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90639

906390

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 Calculus, 2009

90639 Sketch graphs of conic sections and write equations related to conic sections.

Credits: Three
2.00 pm Thursday 26 November 2009

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–19 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**				
Achievement		**Achievement with Merit**		**Achievement with Excellence**	
Sketch graphs of conic sections and write equations related to conic sections.		Solve problems involving conic sections.		Solve more complex conic section problems.	
Overall Level of Performance					

You are advised to spend 40 minutes answering the questions in this booklet.

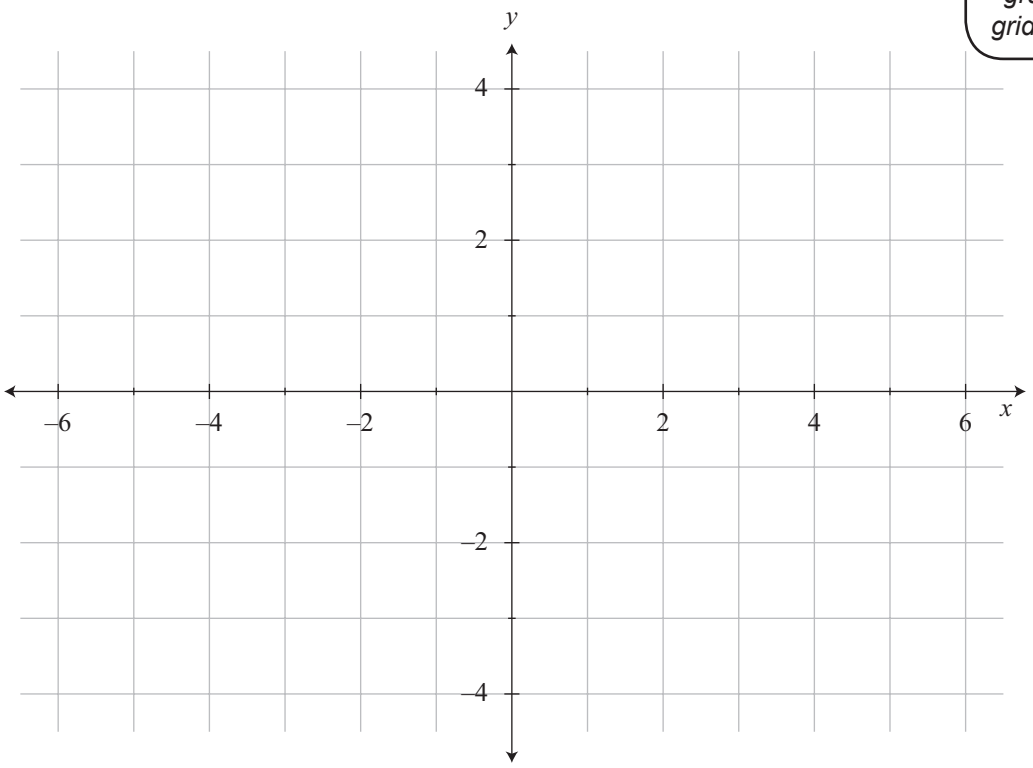
Assessor's use only

QUESTION ONE

(a) Sketch the graph of the curve defined by $x = 4\cos\theta$ and $y = \sin\theta + 1$.

Label any intercepts and asymptotes.

If you need to redraw this graph, use the grid on page 14.

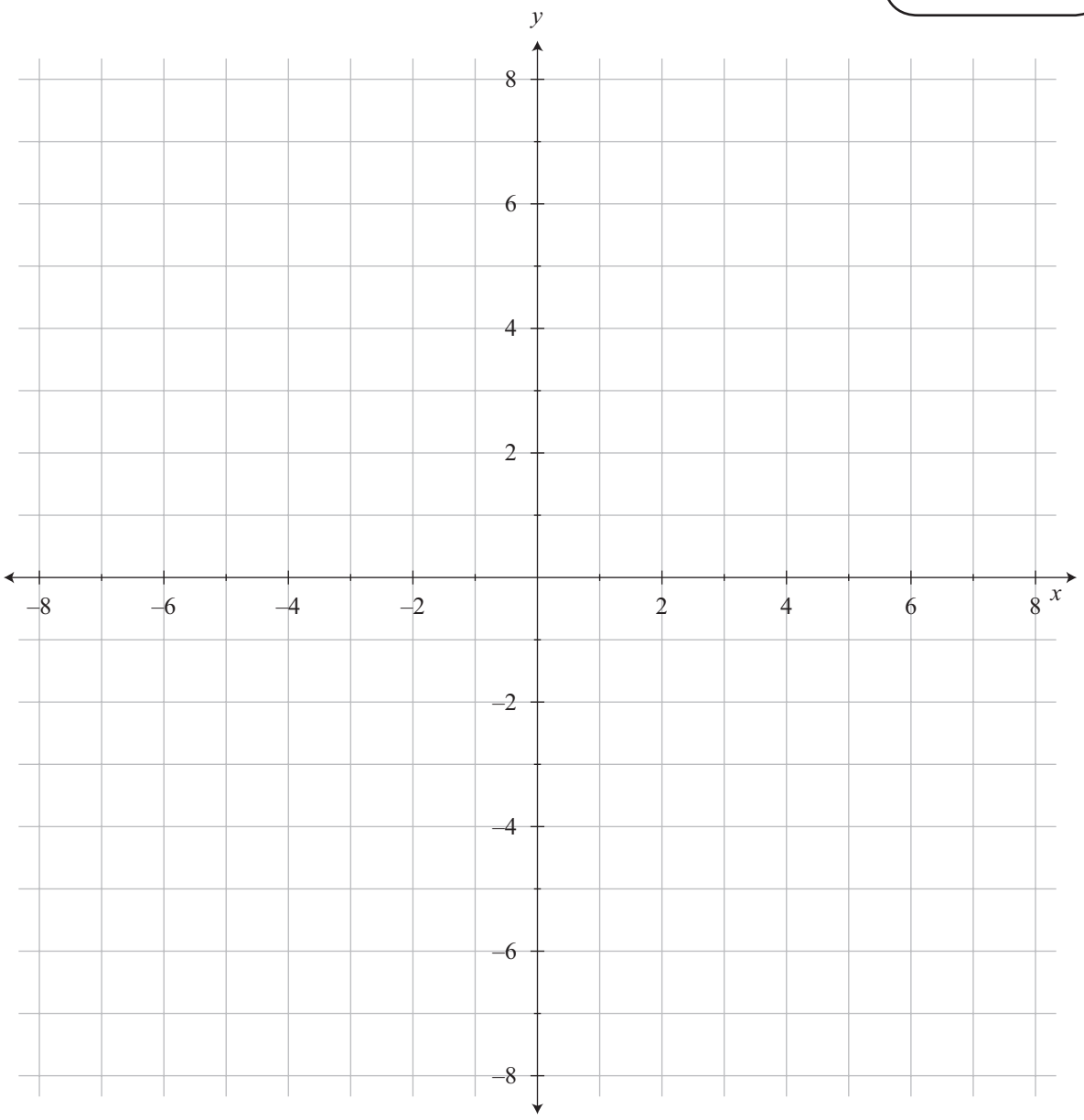


(b) Sketch the graph of $9x^2 - 16y^2 = 144$.

Label any intercepts and asymptotes.

If you need to redraw this graph, use the grid on page 15.

Assessor's use only



(c) Write the equation of the conic section shown below.

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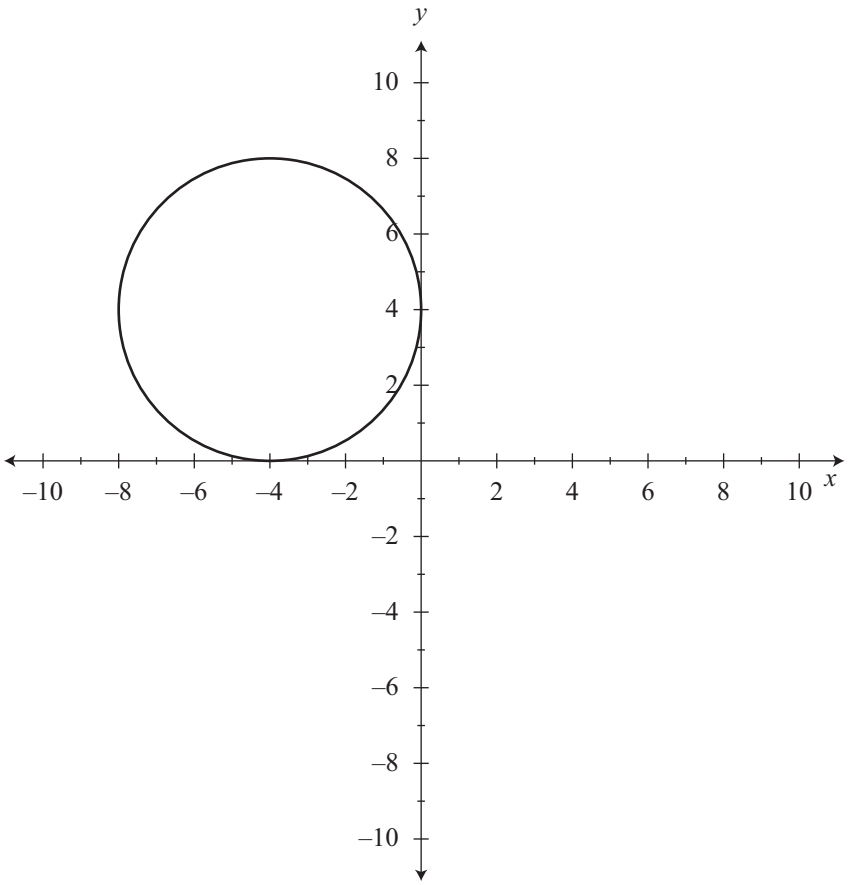

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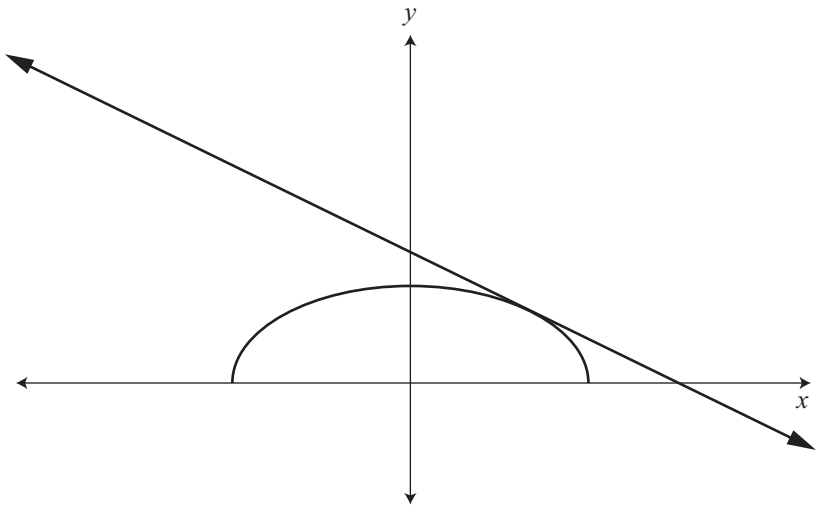
(d) An ellipse has the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

The upper half of the ellipse is drawn in the diagram below.

A tangent is drawn at $x = 2$.



Find the gradient of the tangent.

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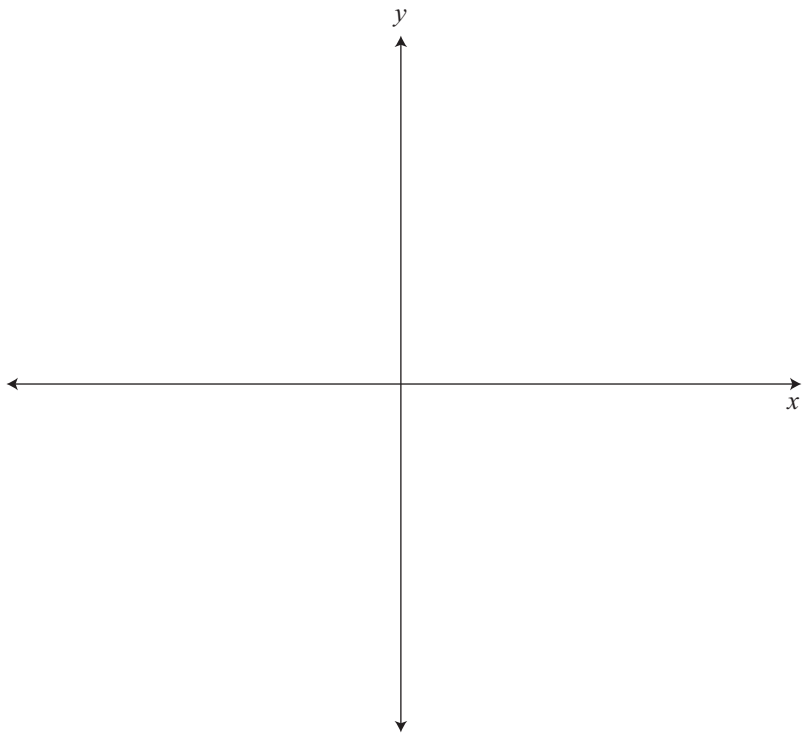
(e) An ellipse has the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b > 0$.

A tangent to the ellipse is drawn at (m,n) in the upper half of the ellipse.
The tangent cuts the *x*-axis at (c,0).

Show, by finding the gradient of the tangent or otherwise, that

$$\frac{a^2}{b^2} = \frac{mc - m^2}{n^2}$$

Hint: You may find it helpful to sketch a graph.



If you need to redraw this graph, use the grid on page 16.

Assessor's use only

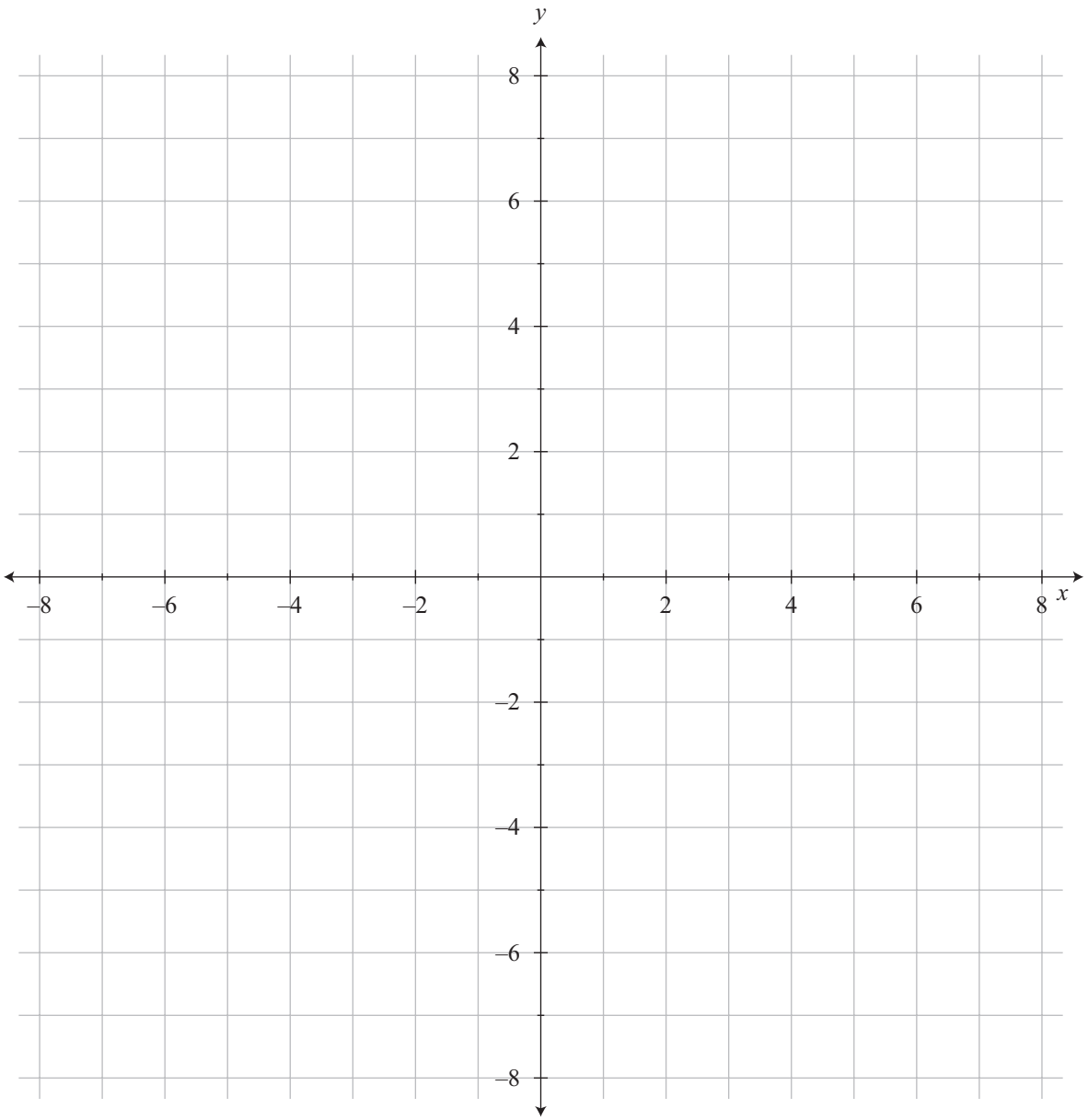
QUESTION TWO

If you need to redraw this graph, use the grid on page 17.

Assessor's use only

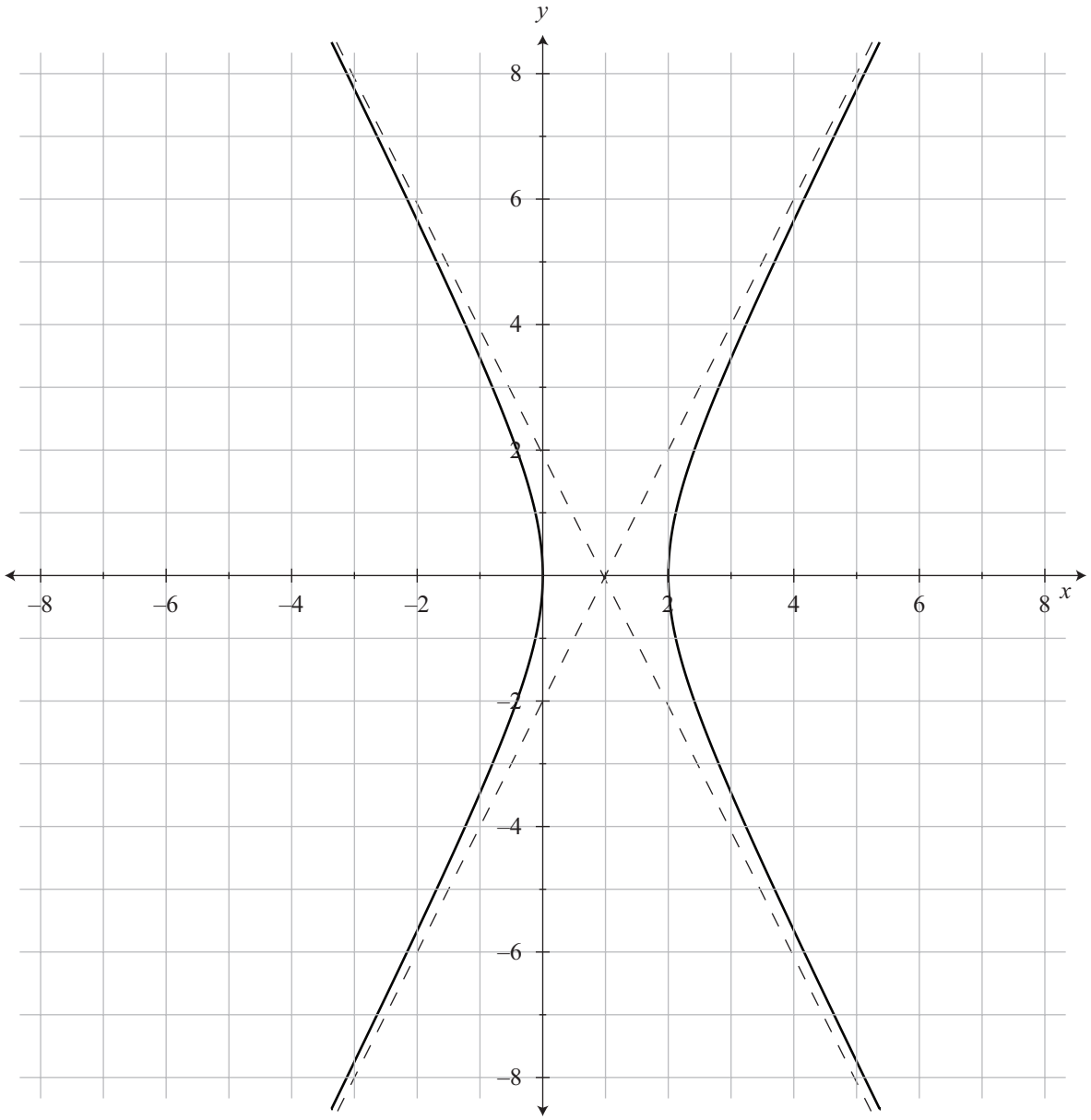
(a) Sketch the graph of the curve defined by $y = 4t$ and $x = 2t^2 - 2$.

Label any intercepts and asymptotes.



(b) Write the equation of the conic section shown below.

Assessor's use only



(c) The ellipse shown below is centred at (0,0) and is symmetrical about the x and y axes.
Point E is at one focus of the ellipse.
The distance from point E to point P is 4 units.
The distance from point E to point Q is 44 units.

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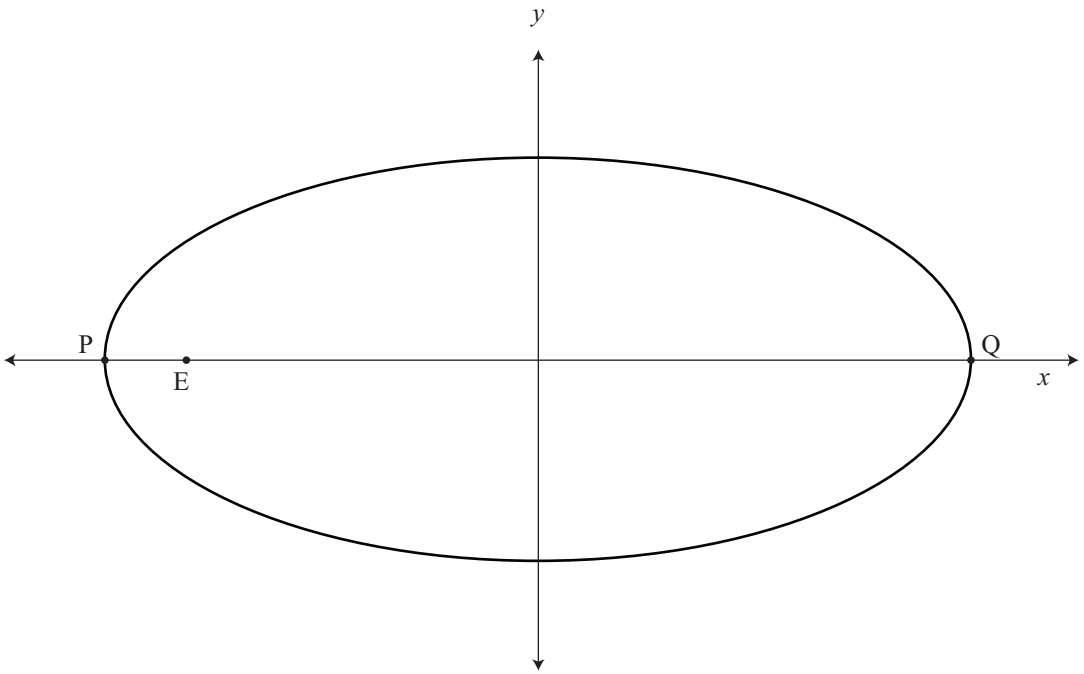

Find the equation of the ellipse.

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(d) The path of a comet can be modelled by a parabola with equation $y^2 = 2x$.

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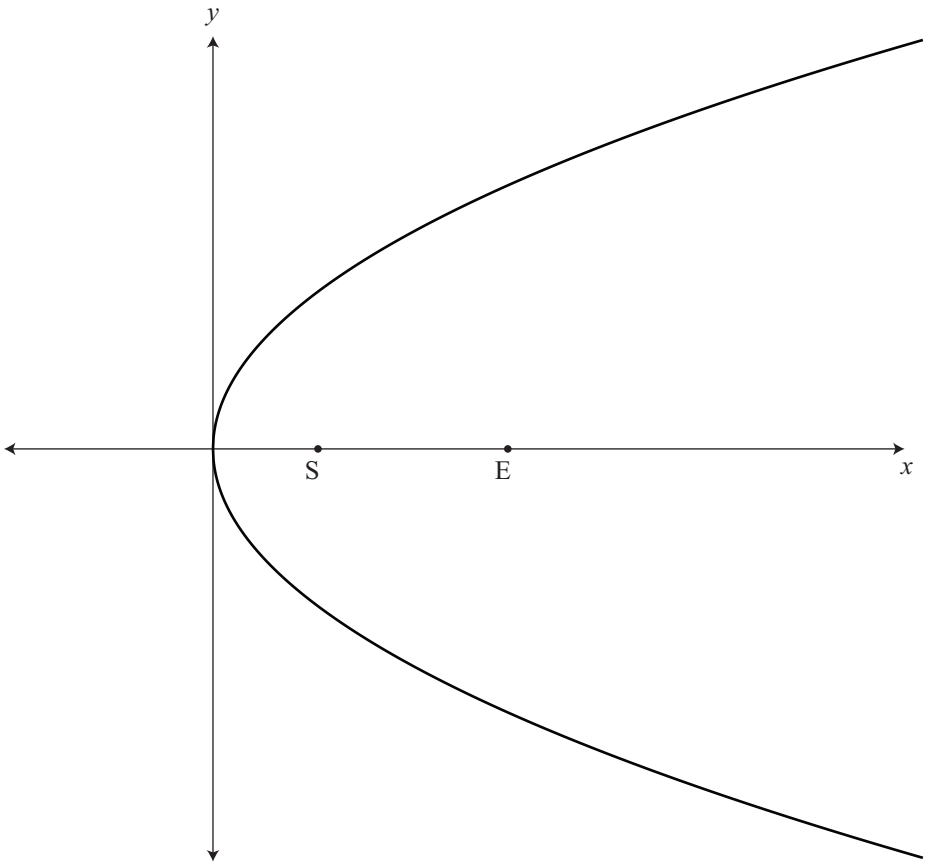
The Sun, at S, is at the focus of the parabola, and the Earth, at E, lies along the axis of symmetry of the parabola at a distance of 1 Astronomical Unit from the Sun.



What is the closest the comet gets to the Earth?

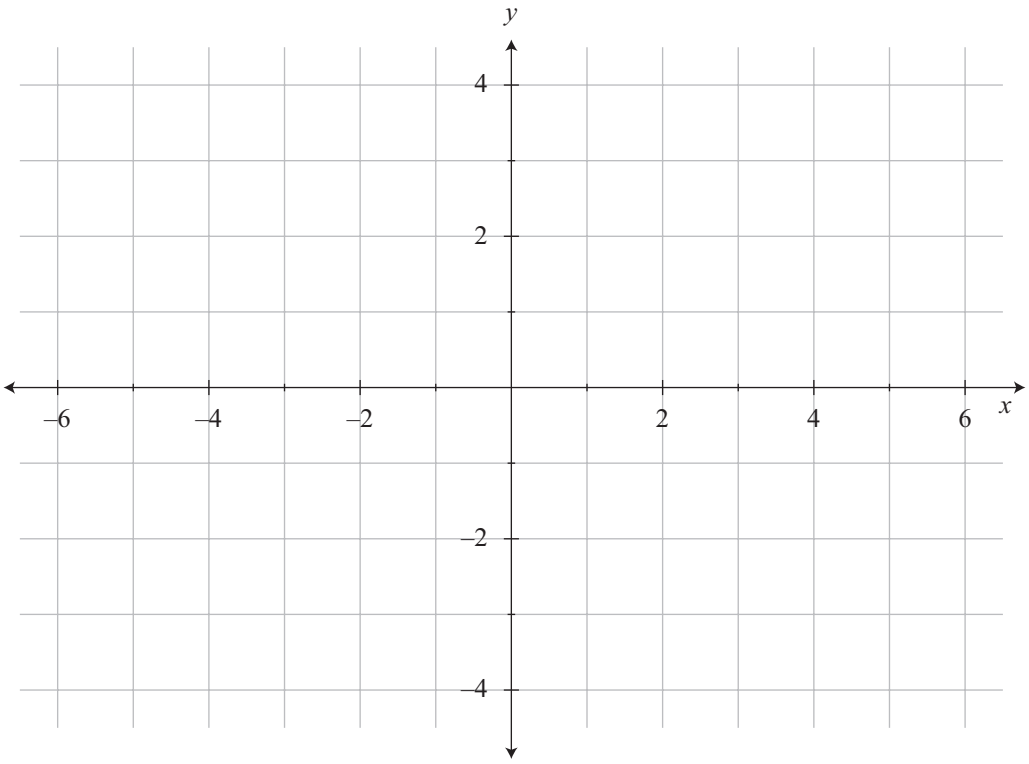
Notes: The movement of the Sun and the Earth can be ignored.
An Astronomical Unit (AU) is the average distance from the Earth to the Sun.

Assessor's use only

If you need to redraw the graph from page 2, draw it on the grid below and carefully number the question. Make sure it is clear which graph from the question you want marked.

Assessor's use only

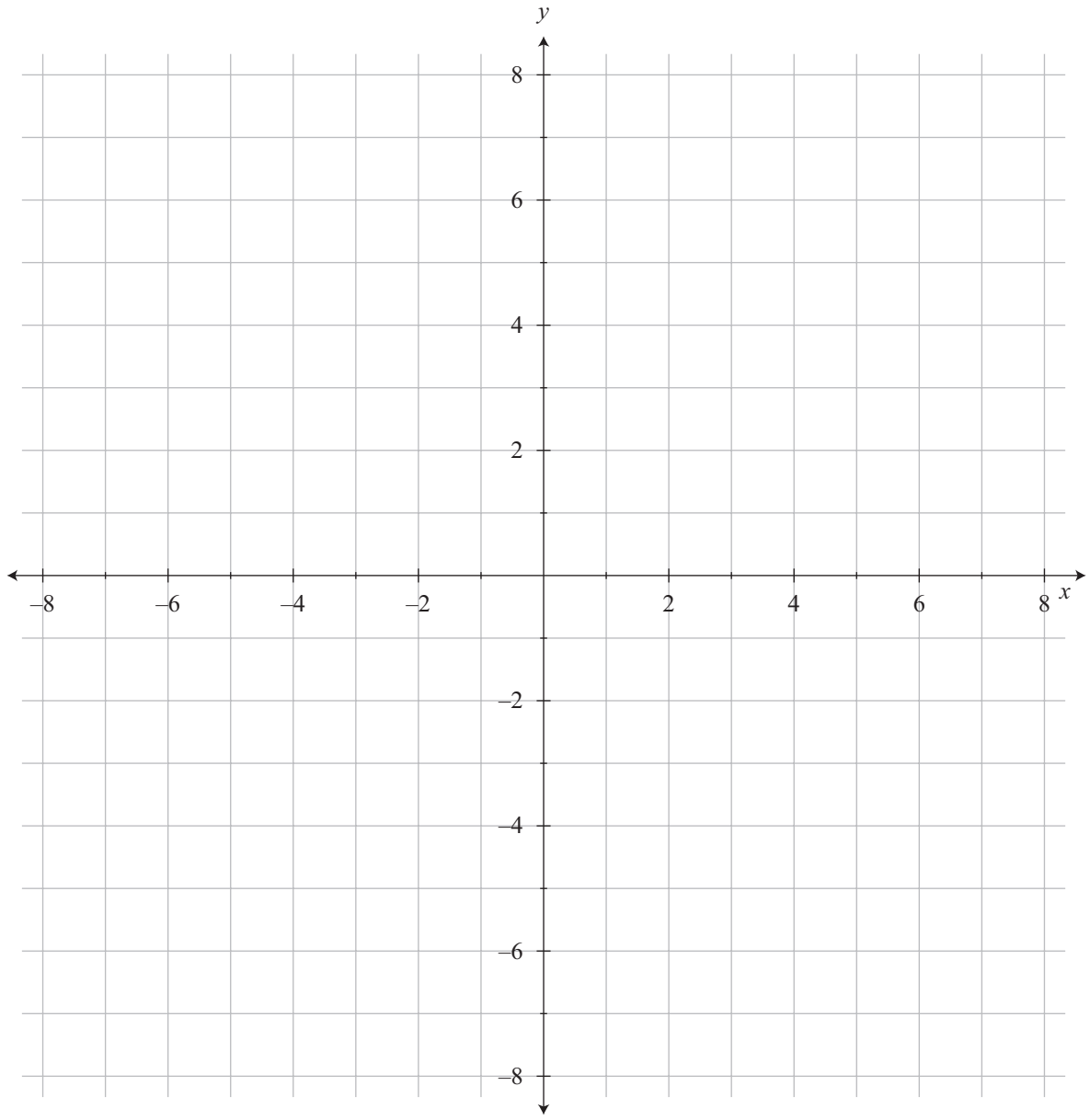
Question _____



If you need to redraw the graph from page 3, draw it on the grid below and carefully number the question. Make sure it is clear which graph from the question you want marked.

Assessor's use only

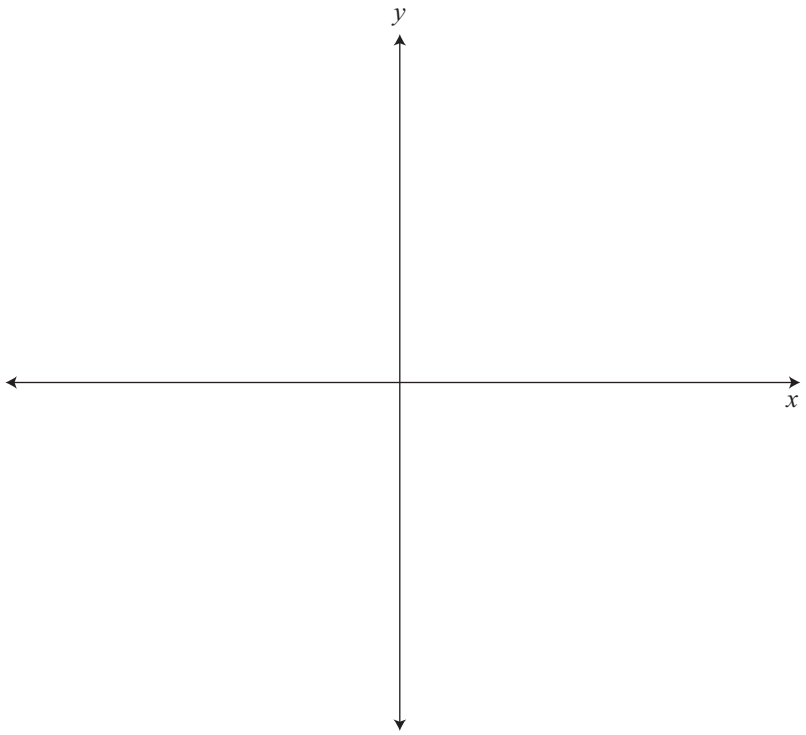
Question _____



If you need to redraw the graph from page 6, draw it on the grid below and carefully number the question. Make sure it is clear which graph from the question you want marked.

Assessor's use only

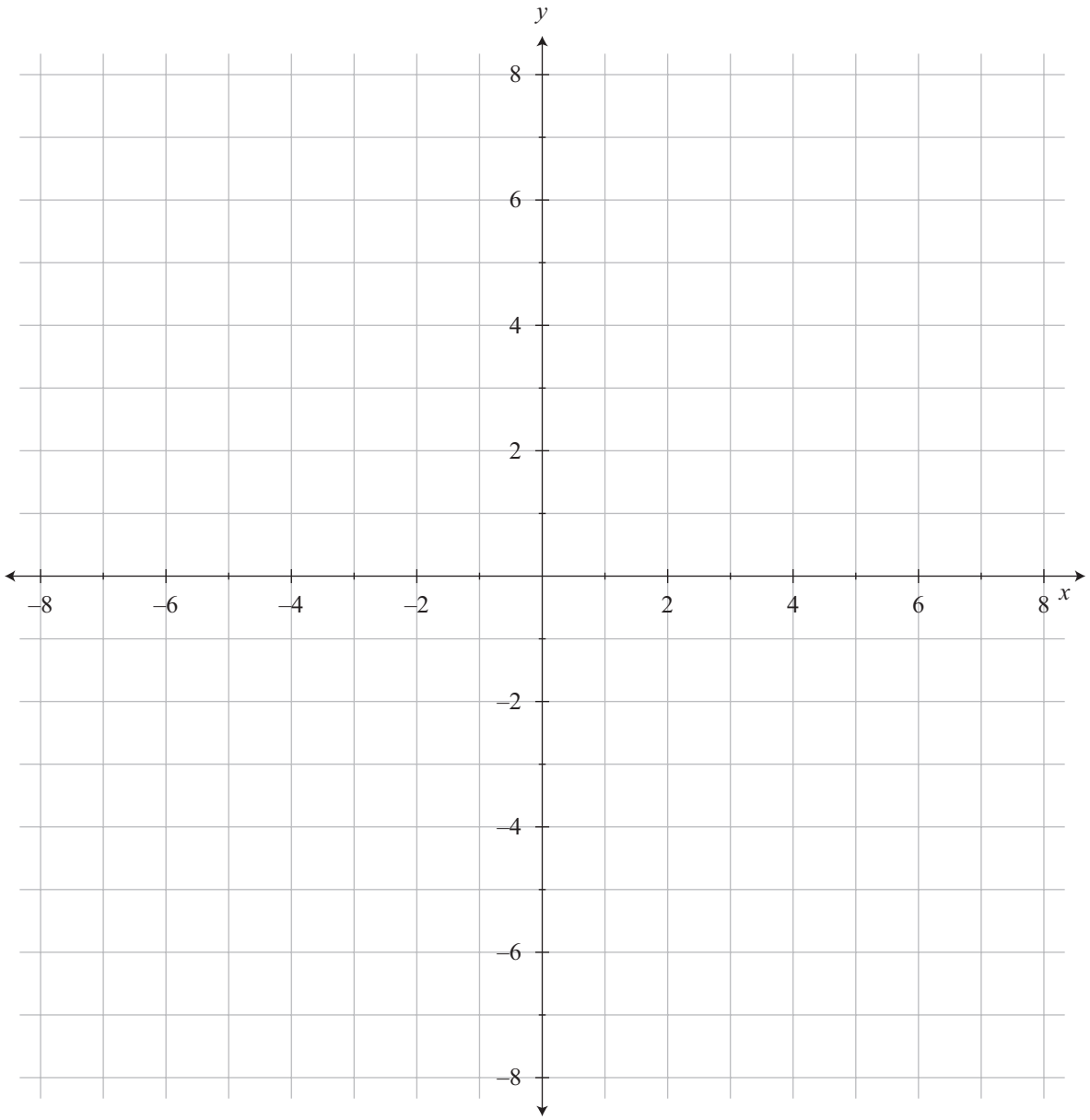
Question _____



If you need to redraw the graph from page 8, draw it on the grid below and carefully number the question. Make sure it is clear which graph from the question you want marked.

Assessor's use only

Question _____



Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number

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